Erratum to
“A Converse to the Second Whitehead Lemma”

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Communicated by W. A. F. Ruppert

Mathematics Subject Classification 2010: 17B56.
Key Words and Phrases: Spectral sequence, Second Whitehead Lemma.

In [6], a result from [5] is used, claiming that the Hochschild–Serre spectral sequence abutting to the cohomology of the semidirect sum \( L = S + I \) of Lie algebras \( S \) and \( I \) (\( S \) is a subalgebra, \( I \) is an ideal, i.e. \( S \) acts on \( I \)) with coefficients in an arbitrary \( L \)-module \( V \), with respect to the ideal \( I \), stabilizes at the \( E_2 \) page ([5, Lemma 1] and [6, Proposition 1.3]). In the whole generality, this statement is false. In fact, as shown in [1], this spectral sequence can stabilize at arbitrarily large step.

This statement is true, however, in many special cases: for example, if the base field is of characteristic zero, \( S \) is a finite-dimensional semisimple Lie algebra, and \( V \) is finite-dimensional ([4, Theorem 13]). More sufficient conditions guaranteeing stabilization at \( E_2 \) can be found in [1, Theorem 3], [2, Corollaries 1.5, 1.6 and Theorem 1.7], and [3, Theorem 1.2].

The main result (Theorem 0.2) of [6] can be salvaged in the following way. The above-mentioned general erroneous claim was used once, in the proof of Lemma 2.1. Lemma 2.1, in its turn, was used in the proof of Theorem 0.2 twice: first time (in the proof of Lemma 2.4) in the situation where \( S \) is semisimple, what is covered by the Hochschild–Serre result mentioned above, and the second time in the situation where \( L = S + I \) is the direct sum of algebras. In the latter case, one can invoke the Kunneth theorem instead of Proposition 1.3.

I am grateful to Donald Barnes who pointed out this error.
References


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Received April 11, 2014
and in final form April 28, 2014